PHOTONIC RADIATIVE CORRECTIONS OF $\mathbf{e^+e^- \to t\bar{t}}$ PROCESS

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Abstract: In this paper we present the QED photonic radiative corrections of the $e^+e^- \to t\bar{t}$ process by using the factorization of ISR and GRACE system as a numerical calculation method. The results exported from GRACE system are compared with the exact $\mathcal{O}(\alpha^2)$ theoretical calculations, and good agreement is found. We consider the beam polarization and the angular distribution as well. The $\delta_{\rm ISR}$ is given to evaluate the effect of ISR. Finally, the deviation between $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha^2)$ is also discussed.

Keywords: radiative corrections, cross section, GRACE system, electron-positron annihilation processes.

1 INTRODUCTION

Radiative corrections are really important for precise prediction of the cross sections for the study on high-energy $e^+e^- \to t\bar{t}$ annihilation processes. Among various corrections from the electro-weak interactions, photonic radiative corrections are known as the corrections to the initial-state particles which give a significant contribution. The final state corrections result just have a small contribution in the total cross section [1], [2].

To investigate the initial-state radiation (ISR) of $e^+e^- \to t\bar{t}$ annihilation process, we use the factorization of ISR as theoretical calculation method. After calculating out the ISR function, we inspect the order $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha^2)$ corrections by re-summed formula. In this study, we focus on the electroweak correction contribution in the case that it is the dominant part where as QED correction is controllable. Thus, we apply the numerical calculation using GRACE system to evaluate the total cross section of all diagrams of $e^+e^- \to t\bar{t}$ process in the center-of-mass energy range from 350 GeV to 1 TeV. The results exported from GRACE system are compared with the exact $\mathcal{O}(\alpha^2)$ theoretical calculations, and good agreement is found. Considering the beam polarization in the next step, using

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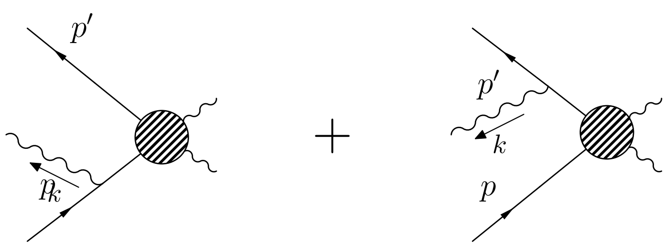
GRACE system, we calculate out the difference between the total cross section as the function of center-of-mass energy of $t\bar{t}$ pair production with the left-handed electron and right-handed positron initial polarization and discuss about the effect of ISR by using the results in ideal polarization case $e^-e^+(100\%, 100\%)$ and in reality polarization case $e^-e^+(80\%, 30\%)$ as well. The effect of ISR on the angular distribution is also important and need to be investigated. After that, we consider the effect of ISR correction by using the formula $\delta_{\text{ISR}} = (\sigma_{\text{ISR}} - \sigma_{\text{No ISR}})/\sigma_{\text{No ISR}}$. Finally, we discuss the deviation between the total cross section in order $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha^2)$ at 500 GeV and 800 GeV center-of-mass energy.

2 CALCULATION METHOD

Firstly, we use the factorization method to calculate the ISR function. The process in which one photon is radiated during the scattering of an electron has the diagrams as bellow



Let $\mathcal{M}_0$ denote the part of the amplitude that comes from the electron's interaction with the external field. Using Feynman rules, the amplitude for the whole process is

$$|\mathcal{M}|^2 \;=\; |\mathcal{M}_1 + \mathcal{M}_2|^2 = \mathcal{M}_1\mathcal{M}_1^\dagger + \mathcal{M}_2\mathcal{M}_2^\dagger + 2\mathcal{M}_1\mathcal{M}_2^\dagger. \tag{1}$$

In the soft-photon approximation, the squared amplitude becomes

$$|\mathcal{M}|^2 = e^2\left[\frac{(p\cdot\epsilon^*)^2}{(p\cdot k)^2} + \frac{(p'\cdot\epsilon^*)^2}{(p'\cdot k)^2} - \frac{2(p\cdot\epsilon^*)(p'\cdot\epsilon^*)}{(p\cdot k)(p'\cdot k)}\right]|e\mathcal{M}_0|^2. \tag{2}$$

Insert an additional phase-space integration for the photon variable k, we have the cross section for our process in terms of the elastic cross section

$$d\sigma(p\to p'+\gamma) = d\sigma(p'\to p)\cdot\int\frac{d^3k}{(2\pi)^3}\frac{1}{2k_0}\sum_{\lambda=1,2}e^2\left|\frac{p'\cdot\epsilon^{(\lambda)}}{p'\cdot k} - \frac{p\cdot\epsilon^{(\lambda)}}{p\cdot k}\right|^2. \tag{3}$$

Expand the absolute sign term and plug the results we obtained back to Eq. (3), we have

$$\text{Total probability} \approx \frac{\alpha}{\pi}\int_0^{|\mathbf{q}|} dk_0\frac{1}{k_0}\mathcal{I}(\mathbf{v},\mathbf{v}'), \tag{4}$$

where

$$\mathcal{I}(\mathbf{v},\mathbf{v}') = \int \frac{d\Omega_{\hat{k}}}{4\pi}\left(\frac{2(1-\mathbf{v}\cdot\mathbf{v}')}{(1-\hat{k}\cdot\mathbf{v})(1-\hat{k}\cdot\mathbf{v}')} - \frac{m^2/E^2}{(1-\hat{k}\cdot\mathbf{v}')^2} - \frac{m^2/E^2}{(1-\hat{k}\cdot\mathbf{v})^2}\right). \tag{5}$$

Since $\mathcal{I}(\mathbf{v},\mathbf{v}')$ is dependent of k, the integral diverges at its lower limit. This is the famous problem of infrared divergences in QED perturbation theory. We can artificially make the integral in (4) well-defined by pretending that the photon has a very small mass μ. This mass would then provide a lower cutoff for the integral, allowing us to write the result of soft Bremsstrahlung quantum computation as

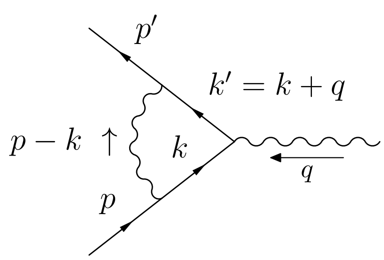
$$\begin{aligned} d\sigma\big(p\to p'+\gamma(k)\big) &= d\sigma(p\to p')\cdot\frac{\alpha}{2\pi}\log\left(\frac{-q^2}{\mu^2}\right)\mathcal{I}(\mathbf{v},\mathbf{v}') \\ &\overset{-q^2\to\infty}{\approx} d\sigma(p\to p')\cdot\frac{\alpha}{\pi}\log\left(\frac{-q^2}{\mu^2}\right)\log\left(\frac{-q^2}{m^2}\right), \end{aligned} \tag{6}$$

where

$$\frac{\alpha}{\pi}\int_0^{|\mathbf{q}|} dk_0 \frac{1}{k_0}\mathcal{I}(\mathbf{v},\mathbf{v}') = \frac{\alpha}{\pi}\mathcal{I}(\mathbf{v},\mathbf{v}')\log\left(\frac{-q^2}{\mu^2}\right). \tag{7}$$

The q^2 dependence of this result, known as *Sudakov double logarithm*, is physical. However, the dependence on μ presents a problem that we must solve in the next steps.

Next, we evaluate the electron vertex function. Assign momenta on the diagram as follows:



Applying the Feynman rules, we find that $\Gamma^\mu = \Gamma^\mu + \delta\Gamma^\mu$ to order α, where

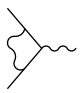
$$\bar{u}(p')\delta\Gamma^\mu(p',p)u(p) = 2ie^2\int\frac{d^4k}{(2\pi)^4}\frac{\bar{u}(p')[\not{k}\gamma^\mu\not{k}' + m^2\gamma^\mu - 2m(k+k')^\mu]u(p)}{\big((k-p)^2+i\epsilon\big)(k'^2-m^2+i\epsilon)(k^2-m^2+i\epsilon)}. \tag{8}$$

Using the contraction identity $\gamma^\nu\gamma^\mu\gamma_\nu = -2\gamma^\mu$ and the "*Feynman parameters*", our expression for the $\mathcal{O}(\alpha)$ contribution to the electron vertex then becomes

$$\begin{aligned} \bar{u}(p')\delta\Gamma^\mu(p',p)u(p) = 2ie^2 &\int\frac{d^4\ell}{(2\pi)^4}\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\frac{2}{D^3} \\ &\times \bar{u}(p')\big[\gamma^\mu\cdot\big(-\tfrac{1}{2}\ell^2 + (1-x)(1-y)q^2 + (1-4z+z^2)m^2\big) \\ &+ \frac{i\sigma^{\mu\nu}q_\nu}{2m}\big(2m^2z(1-z)\big)\big]u(p), \end{aligned} \tag{9}$$

where as before, $D = \ell^2 - \Delta + i\epsilon$, $\Delta = -xyq^2 + (1-z)^2 m^2 > 0$. The decomposition into form factors is now manifest.

Making use of the Dirac equation and a trick called "*Wick rotation*", we get the expression for the one-loop vertex correction



$$= \frac{\alpha}{2\pi} \int_0^1 dx\, dy\, dz\, \delta(x+y+z-1) \times \bar{u}(p') \Bigg(\gamma^\mu \left[\log \frac{z\Lambda^2}{\Delta} + \frac{1}{\Delta}\big((1-x)(1-y)q^2 + (1-4z+z^2)m^2\big) \right] + \frac{i\sigma^{\mu\nu} q_\nu}{2m} \left[\frac{1}{\Delta} 2m^2 z(1-z) \right] \Bigg) u(p).$$

The bracketed expressions are our desired corrections to the form factors. The form factor we need is

$$\begin{aligned} F_1(q^2) =& 1 + \frac{\alpha}{2\pi} \int_0^1 dx\, dy\, dz\, \delta(x+y+z-1) \times \left[\log \left(\frac{m^2(1-z)^2}{m^2(1-z)^2 - q^2 xy} \right) \right. \\ &\left. + \frac{m^2(1-4z+z^2) + q^2(1-x)(1-y)}{m^2(1-z)^2 - q^2 xy + \mu^2 z} - \frac{m^2(1-4z+z^2)}{m^2(1-z)^2 + \mu^2 z} \right] + \mathcal{O}(\alpha^2). \end{aligned} \tag{10}$$

Now let us confront the infrared divergence in our result (10) for $F_1(q^2)$. The dominant part in the $\mu \to 0$ limit, is

$$\begin{aligned} F_1(q^2) \approx& \frac{\alpha}{2\pi} \int_0^1 dx\, dy\, dz\, \delta(x+y+z-1) \\ &\times \left[\frac{m^2(1-4z+z^2 + q^2(1-x)(1-y)}{m^2(1-z)^2 - q^2 xy + \mu^2 z} - \frac{m^2(1-4z+z^2)}{m^2(1-z)^2 + \mu^2 z} \right]. \end{aligned} \tag{11}$$

The form factor in the limit $-q^2 \to \infty$ is

$$F_1(q^2 \to \infty) = 1 - \frac{\alpha}{2\pi} \log\left(\frac{-q^2}{m^2} \right) \log\left(\frac{-q^2}{\mu^2} \right) + \mathcal{O}(\alpha^2). \tag{12}$$

If the same relation holds for general q^2, the measured cross section becomes

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{measured}} \approx \left(\frac{d\sigma}{d\Omega} \right)_0 \left[1 - \frac{\alpha}{\pi} f_{\text{ISR}}(q^2) \log\left(\frac{-q^2 \text{ or } m^2}{E_\ell^2} \right) + \mathcal{O}(\alpha^2) \right]. \tag{13}$$

Recall from Eq. (12), however, that we were careful to obtain the correct coefficient of $\log^2(-q^2)$ in the limit $-q^2 \gg m^2$. In that limit, therefore, Eq. (13) becomes

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{measured}} \approx \left(\frac{d\sigma}{d\Omega} \right)_0 \left[1 - \frac{\alpha}{\pi} \log\left(\frac{-q^2}{m^2} \right) \log\left(\frac{-q^2}{E_\ell^2} \right) + \mathcal{O}(\alpha^2) \right]. \tag{14}$$

Considering the $\mathcal{O}(\alpha^2)$ in total cross section calculation with ISR correction, we have $\sigma_{\text{total}} = \int\limits_0^1 dx H(x,s)\sigma_0\big(s(1-x)\big)$ [3], where

$$H(x,s) = \Delta \cdot \beta x^{\beta-1} - \Delta_1\beta\left(1-\frac{x}{2}\right) + \frac{\beta^2}{8}\left[-4(2-x)\ln x - \frac{1+3(1-x)^2}{x}\ln(1-x) - 2x\right], \tag{15}$$

with $\Delta = 1 + \Delta\delta^{(1)} + \Delta\delta^{(2)}$ and $\Delta_1 = 1 + \Delta\delta^{(1)}$, here $\Delta\delta^{(1)}$ is a part of the lowest order corrections and so is $\Delta\delta^{(2)}$ of the second order corrections. Their formulas are given by

$$\Delta\delta^{(1)} = \frac{\alpha}{\pi}\left(\frac{3}{2}L + \frac{\pi^2}{3} - 2\right) \text{ and } \Delta\delta^{(2)} \simeq \left(\frac{\alpha}{\pi}\right)^2 (L-1)^2\left(\frac{9}{8} - \frac{\pi^2}{3}\right).$$

Contemplating the $\mathcal{O}(\alpha)$ of ISR correction in total cross section calculation, we have $\sigma_{\text{total}} = \int\limits_0^1 dx H(x,s)\sigma_0\big(s(1-x)\big)$, where $H(x,s) = \Delta \cdot \beta x^{\beta-1} - \Delta_1\beta(1-x/2)$ with $\Delta = 1 + \Delta\delta^{(1)} + \Delta\delta^{(2)}$, $\Delta_1 = 1 + \Delta\delta^{(1)}$ and $\beta = \frac{2\alpha}{\pi}\left(\ln\frac{s}{m^2} - 1\right)$ [3], [4].

3 RESULTS AND DISCUSSIONS

Our main target is calculate the cross section of $e^+e^- \to t\bar{t}$ process at the energy region from 350 GeV to 1 TeV with all diagrams contribution as Fig. 1.

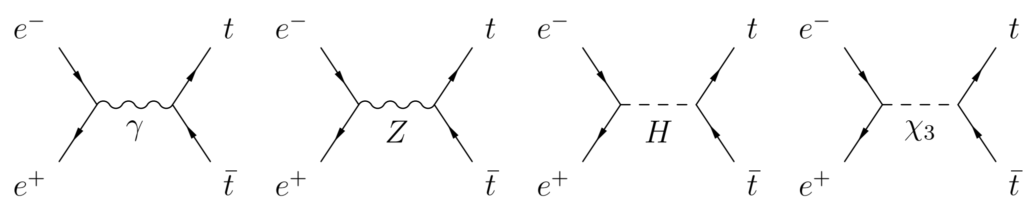


Fig. 1: All diagrams for $e^+e^- \to t\bar{t}$ process at tree level.

Based on the data we get from GRACE system, we plot Fig. 2. This figure represents the cross section distribution of $e^+e^- \to t\bar{t}$ process with all diagrams contribution. With ISR effect, after emitting photons, the cross section distributions has been shifted to two parts, the left handed side part is now below and the right handed side part is now above the cross section distribution in case without ISR. This effect exists because of the behavior of f_{ISR} function as Eq. (15). The energy of emitting photons is $E_\ell = \sqrt{s}x$ and the total energy after emitting photon is $\sqrt{s'} = q(1-x) \to E_\ell + \sqrt{s'} = \sqrt{s}$. We infer

$$f_{\text{ISR}} \propto \ln\frac{E_\ell^2}{s}. \tag{16}$$

To contemplate the e^+e^- beam polarization, we present the difference between the total cross section as the function of CM energy of $t\bar{t}$ pair production with the left-handed electron and right-handed positron initial polarization.

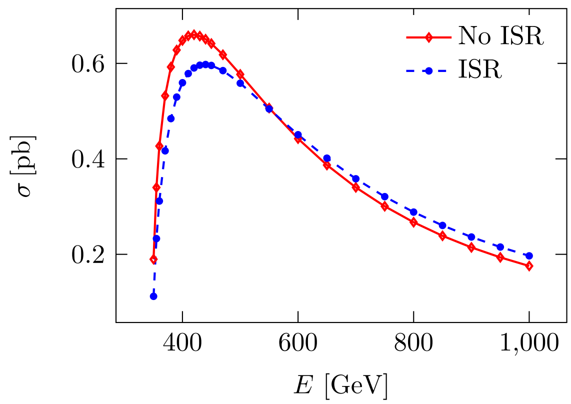


Fig. 2: Total cross section of $e^+e^- \to t\bar{t}$ process from 350 GeV to 1 TeV center of mass energy with all diagrams contribution.

Based on the results of total cross section of non polarization, $e^-_{\mathrm{L}}e^+_{\mathrm{R}}$ (100%, 100%) and $e^-_{\mathrm{R}}e^+_{\mathrm{L}}$ (100%, 100%) in case without and with ISR, we plot Fig. 3.

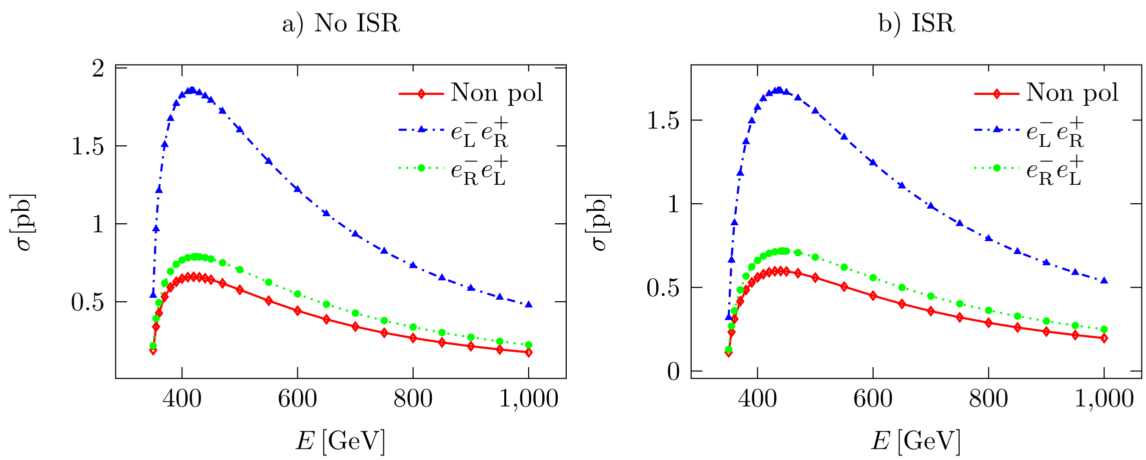


Fig. 3: Total cross section of $e^+e^- \to t\bar{t}$ process from 350 GeV to 1 TeV center of mass energy in polarization cases e^-e^+ $(100\%, 100\%)$ and non polarization case without and with ISR effect.

In these figures, the distribution of the cross section of $e^-_{\mathrm{L}}e^+_{\mathrm{R}}$ is highest, the distribution of $e^-_{\mathrm{R}}e^+_{\mathrm{L}}$ is in the middle and the distribution in non polarization case is lowest. The reasons of this behavior are because of the distributions of $e^-_{\mathrm{L}}e^+_{\mathrm{L}}$ and $e^-_{\mathrm{R}}e^+_{\mathrm{R}}$ are almost zero and the distribution of non polarization case is the average of all four cases. Thus, this distribution must be lower than the distributions in $e^-_{\mathrm{L}}e^+_{\mathrm{R}}$ and $e^-_{\mathrm{R}}e^+_{\mathrm{L}}$ beam polarization cases. The effect of ISR just shifts the distributions but remains the relative positions of them.

Investigating the electron beam polarization in the real experiment, based on the results of total cross section of non polarization, $e^-_{\mathrm{L}}e^+_{\mathrm{R}}$ (80%, 30%) and $e^-_{\mathrm{R}}e^+_{\mathrm{L}}$ (80%, 30%) in case without and with ISR, we plot Fig. 4.

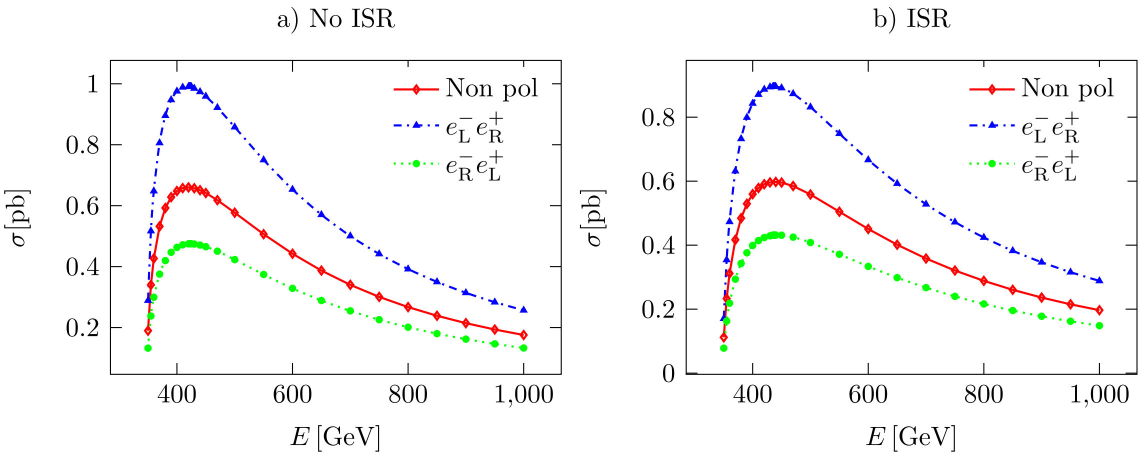


Fig. 4: Total cross section of $e^+e^- \to t\bar{t}$ process from 350 GeV to 1 TeV center of mass energy in polarization cases e^-e^+ $(80\%, 30\%)$ and non polarization case without and with ISR effect.

This figure represents the total cross section of $e^+e^- \to t\bar{t}$ process from 350 GeV to 1 TeV center of mass energy in polarization cases e^-e^+ (80%, 30%) and non polarization case without and with ISR effect. We can see that the ratio of $e^-_{\mathrm{L}}e^+_{\mathrm{R}}$ and $e^-_{\mathrm{R}}e^+_{\mathrm{L}}$ have been changed. Hence, the distributions position are changed. The effect of ISR just shifts the distributions but remains the relative positions of them.

To investigate the angular distribution, we calculate the total cross section for the following process: $e^-_{\mathrm{L}}e^+_{\mathrm{R}} \to t\bar{t}$ and $e^-_{\mathrm{R}}e^+_{\mathrm{L}} \to t\bar{t}$. In our calculation, processes involving the $e^-_{\mathrm{L}}e^+_{\mathrm{L}}$ and $e^-_{\mathrm{R}}e^+_{\mathrm{R}}$ combinations are omitted because they yield negligible contribution.

We now consider the angular distribution of $e^+e^- \to t\bar{t}$ process at 500 GeV colliding energy. Exporting the data in GRACE system, we obtain enough data to plot Fig. 5.

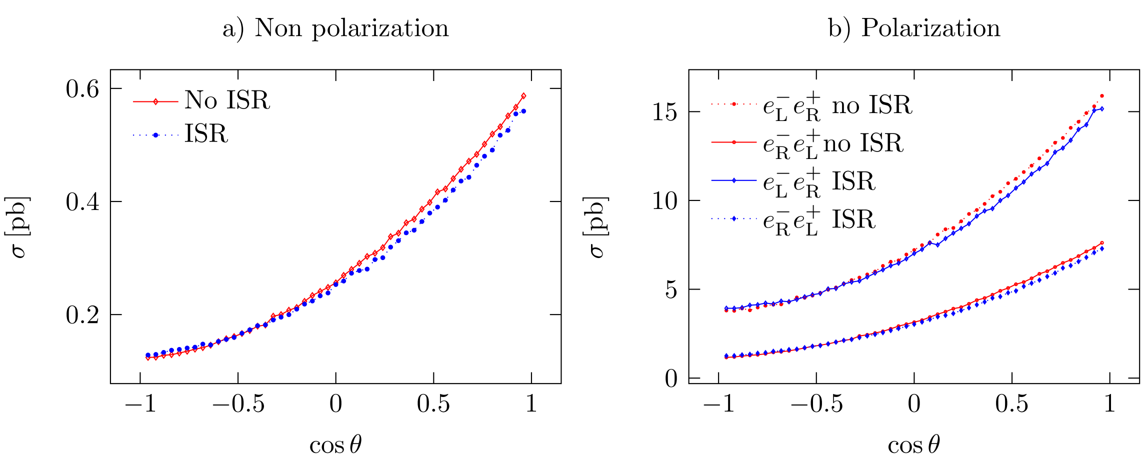


Fig. 5: Angular distribution of $e^+e^- \to t\bar{t}$ process with beam polarization at 500 GeV center of mass energy.

In Fig. 5 a), with ISR, there are some probabilities of changing angular distribution which effect the energy of electron and positron, hence the angular distributions are shifted. Therefore, we see the left handed side distribution with ISR effect are below the distribution of no ISR case while the right handed side distribution with ISR effect are above the distribution of no ISR case. In Fig. 5 b), we can see that the relative position between the angular distributions in cases effected by ISR effect and in cases without ISR effect are still the same.

Using GRACE system to calculate the total cross section of $e^+e^- \to t\bar{t}$ process from 350 GeV to 1 TeV center of mass energy in case without and with ISR. Then we use numerical analysis method to handle the data and get the $\delta_{\rm ISR}$. Based on the results in non polarization and polarization $e^-_{\rm L}e^+_{\rm R}(80\%, 30\%)$ cases, we plot Fig. 6.

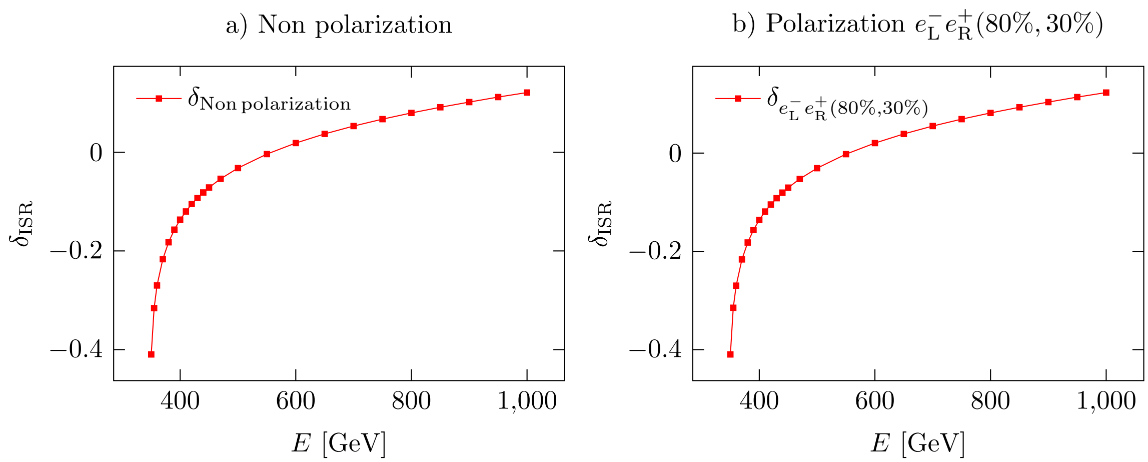


Fig. 6: $\delta_{\rm ISR}$ of $e^+e^- \to t\bar{t}$ process from 350 GeV to 1 TeV center of mass energy in non polarization and beam polarization $e^-_{\rm L}e^+_{\rm R}(80\%, 30\%)$.

We clearly see that the results are exactly the same. Thus, in conclusion, the ISR does not effect on beam polarization $e^+e^- \to t\bar{t}$ process.

Using GRACE system to calculate the total cross section of $e^+e^- \to t\bar{t}$ at 500 GeV and 800 GeV center of mass energy. At 500 GeV, we see that the total cross section in case with ISR $\mathcal{O}(\alpha)$ is about 5% larger than the total cross section in case with ISR $\mathcal{O}(\alpha^2)$. At 800 GeV the total cross section in case with ISR $\mathcal{O}(\alpha)$ is about 10% larger than the total cross section in case with ISR $\mathcal{O}(\alpha^2)$. Based on these results, we can conclude that the $\mathcal{O}(\alpha^2)$ has a very important effect on ISR correction. It represents how precise ISR $\mathcal{O}(\alpha^2)$ can correct our results while study about $e^+e^- \to t\bar{t}$ process.

4 CONCLUSIONS

In this paper, we have studied on the photonic radiative corrections of $e^+e^- \to t\bar{t}$ process with and without beam polarization. Besides, we considered the angular distribution, the δ_{ISR}, the $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha^2)$ as well. We used the Quantum Field Theory to calculate out the formula of the photonic radiative correction and applied it onto the GRACE system for $e^-e^+ \to t\bar{t}$ process at the energy region from 350 GeV to 1 TeV. The results we got from GRACE system were compared with the correspondence with the perturbative calculation of the $\mathcal{O}(\alpha^2)$, and good agreements were found. Based on the results exported from GRACE system in $e^-e^+(100\%, 100\%)$ polarization case, we concluded that the effect of ISR just shifted the cross section distributions but it still remained the corresponding positions of them. Similarly, in $e^-e^+(80\%, 30\%)$ polarization case, the effect of ISR had the same behavior. Furthermore, we concluded that the ISR does not effect on angular distribution of $e^-e^+ \to t\bar{t}$ process by considering the angular distribution. In addition, the ISR does not effect on beam polarization of $e^-e^+ \to t\bar{t}$ process as well. Finally, by investigating the deviation between ISR $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha^2)$, we inferred the significance of the higher order in cross section calculation of $e^-e^+ \to t\bar{t}$ process.

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